Written exam Control Engineering TBKRT05E

Wednesday 29 October 2008, 14:00 to 17:00 uur

Name: Student ID:

Uitgewerkte versie

- This exam has 13 pages with 5 open questions. Check if you have all pages.
- The answers, including the motivation of the answers, should be placed in the answer boxes. Please put your name and student id on all pages and hand it in in one piece.
- If you like, you can hand in additional pages for extra information. Please put your name and student id on all pages.
- You are allowed to use the book, reader, course material (neatly together) and calculator.
- Read the questions carefully.
- To obtain the grade, the laboratory exercises should have been sufficiently graded.
- Good luck!

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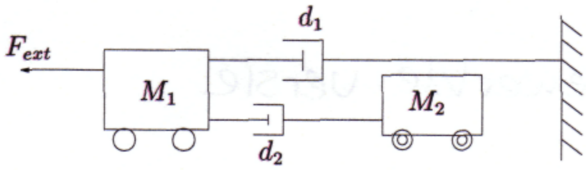
Opgave 1 (~~18~~ 19 punten)

Given a system with 2 masses and 2 dampers, as in the figure below. There is an external force acting on mass 1. The dissipated energy of damper 1 is $\frac{1}{2}d_1 v_1^2$, with v_1 the speed of mass 1. The dissipated energy of damper 2 is $\frac{1}{2}d_2(v_1 - v_2)^2$, with v_2 the speed of mass 2.



a). What is the Lagrangian of the above system?

4 $\mathcal{L}(x,v) = \frac{1}{2}M_1 v_1^2 + \frac{1}{2}M_2 v_2^2$

b). How many kinetic and potential energy storing elements has the above system? Motivate your answer.

Two kinetic energy storing elements (2 masses)

3

No potential energy storing elements.

c). What is de Rayleigh dissipation function of the above system?

3 $D(v) = \frac{1}{2} d_1 v_1^2 + \frac{1}{2} d_2 (v_1 - v_2)^2$

d). Give the Euler-Lagrange equations of the above system.

$$\frac{\partial \mathcal{L}}{\partial v} = \begin{pmatrix} \frac{\partial \mathcal{L}}{\partial v_1} \\ \frac{\partial \mathcal{L}}{\partial v_2} \end{pmatrix} = \begin{pmatrix} M_1 v_1 \\ M_2 v_2 \end{pmatrix} \quad \frac{\partial \mathcal{L}}{\partial x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \frac{\partial D}{\partial v} = \begin{pmatrix} d_1 v_1 + d_2(v_1 - v_2) \\ -d_2(v_1 - v_2) \end{pmatrix}$$

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$$F = \begin{pmatrix} F_{ext} \\ 0 \end{pmatrix}$$

Hence: $\begin{cases} M_1 \dot{v}_1 = -d_1 v_1 - d_2(v_1 - v_2) + F_{ext} \\ M_2 \dot{v}_2 = d_2(v_1 - v_2) \end{cases}$

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e). Determine a state space model based on the Euler-Lagrange equations.

$x_1 = v_1,\ x_2 = v_2,\ u = F_{ext}$

$$\begin{cases} \dot{x}_1 = -\frac{d_1}{M_1}x_1 - \frac{d_2}{M_1}(x_1 - x_2) + \frac{u}{M_1} \\ \dot{x}_2 = \frac{d_2}{M_2}(x_1 - x_2) \end{cases} = \begin{pmatrix} -\frac{d_1}{M_1} - \frac{d_2}{M_1} & \frac{d_2}{M_1} \\ \frac{d_2}{M_2} & -\frac{d_2}{M_2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{M_1} \\ 0 \end{pmatrix} u$$

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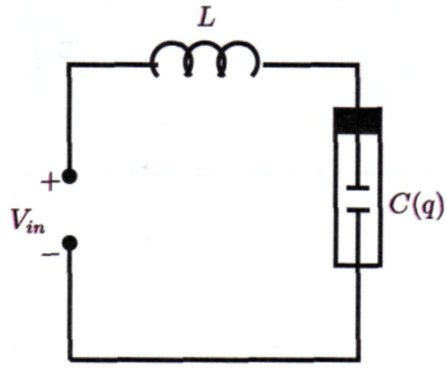
Opgave 2 (~~17~~ 19 punten)

Consider the electrical LC circuit below. The capacity $C(q)$ of the capacitor depends nonlinearly on the charge q. L corresponds to a linear inductor, and the circuit is driven by a controllable voltage source V_{in}. We are interested in the voltage over the condensator, hence that is our output.



The corresponding differential equation is

$$L\ddot{q} + \frac{1}{C_0}q + \frac{4}{C_0}q^3 = V_{in}$$

with q the charge, and with L and C_0 positive constants.
Take the charge q, and the current $\dot{q}$ as the states. With $x_1 = q$, $x_2 = \dot{q}$, $u = V_{in}$ and $y = \frac{1}{C_0}q + \frac{4}{C_0}q^3$ we obtain the following state space system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{1}{LC_0}x_1 - \frac{4}{LC_0}x_1^3 + u \\ y &= \frac{1}{C_0}x_1 + \frac{4}{C_0}x_1^3 \end{aligned}$$

$\Leftarrow \frac{1}{L}u$, niet gerekend.

a). Determine the equilibrium point or points of the system for $u = 0$. Motivate your answer!

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$\dot{x}_1 = 0$, $\dot{x}_2 = 0$ $\Rightarrow$ $x_2 = 0$, $\frac{1}{LC_0}x_1(1+4x_1^2)=0$
$u = 0$
$\Rightarrow x_1 = 0$

Hence, eq. point $(x_1, x_2) = (0,0)$
for $u = 0$

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b). **Linearize the system around a equilibrium point of a). Motivate your answer!**

$$f(x) = \begin{pmatrix} x_2 \\ -\frac{1}{LC_0}x_1 - \frac{u}{LC_0}x_1^3 \end{pmatrix} = \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} \qquad g(x) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$h(x) = \frac{1}{C}x_1 + \frac{u}{C_0}x_1^3 \qquad \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases}$$

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix}\Bigg|_{(0,0)} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{LC_0} - \frac{\cancel{3}\,12}{\cancel{3}LC_0}x_1^2 & 0 \end{pmatrix}\Bigg|_{(0,0)} = \begin{pmatrix} 0 & 1 \\ -\frac{1}{LC_0} & 0 \end{pmatrix}$$

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$$B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = g(0) \qquad C = \begin{pmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \end{pmatrix}\Big|_{(0,0)} = \begin{pmatrix} \frac{1}{C_0} + \frac{\cancel{3}\,12}{\cancel{3}C_0}x_1^2 & 0 \end{pmatrix}\Big|_{(0,0)} = \begin{pmatrix} \frac{1}{C_0} & 0 \end{pmatrix}$$

linearized system:

$$\Delta\dot{x} = A\Delta x + B\Delta u \qquad \Delta x = x - x^*$$
$$\Delta y = C\Delta x \qquad \Delta u = u - u^*$$
$$x^* = (0,0)$$
$$u^* = 0$$

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Opgave 3 (~~18~~ 20 punten)

Given a continuous time state space system with input $u(t)$, output $y(t)$ and $t \in \mathbb{R}$ described by

$$\frac{dx(t)}{dt} = \begin{bmatrix} -2 & -1 \\ 0 & 2 \end{bmatrix} x(t) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} \alpha & 1 \end{bmatrix} x(t)$$

with $\alpha \in \mathbb{R}$, $\alpha \neq 0$, a constant.

a). Is the state space system stable? Motivate your answer! (Hint: eigen values).

3

$A = \begin{pmatrix} -2 & -1 \\ 0 & 2 \end{pmatrix}$ $\quad \det(\lambda I - A) = (\lambda+2)(\lambda-2) = 0$

$\Rightarrow \lambda_1 = -2, \lambda_2 = 2$

$\lambda_2 > 0$, hence unstable!

b). Determine the transfer function of this system.

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$(sI-A)^{-1} = \frac{1}{(s+2)(s-2)} \begin{pmatrix} s-2 & -1 \\ 0 & s+2 \end{pmatrix}$ $\quad C = (\alpha \ 1)$, $B = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

$(sI-A)^{-1}B = \frac{1}{(s+2)(s-2)} \begin{pmatrix} -s+2 \\ 0 \end{pmatrix}$

$H(s) = C(sI-A)^{-1}B = \frac{\alpha(-s+2)}{(s+2)(s-2)}$

c). Determine the poles and zeros of the system. Can you draw a conclusion from this?

4

zeros: $s=2$ $\quad$ poles: $s=-2, s=2$ $\Big\}$ pole-zero cancellation.

Or:

zeros: none $\quad$ poles: $s=-2$ $\Big\}$ first order system.

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d). Is the transfer function stable? Motivate your answer!

3

Yes, $s=-2$ is only pole, $Re(s)<0$, hence as. stable (due to pole-zero cancellation)

e). How many states do you minimally need for the state space system connected to the transfer function (or in other words: what is the order)? Motivate your answer!

3

The order is 1, i.e., only 1 state is needed.

f). What happens if $\alpha = 0$? What does this mean for the influence of the input to the output? Motivate your answer!

3

If $\alpha=0$, then $H(s)=0$, i.e., the input u cannot influence the output y.

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Opgave 4 (20 punten)

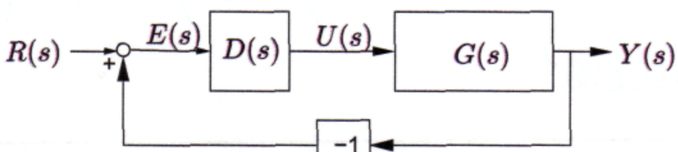
Given is the closed loop system of a gasoline motor described as in figure 1. The following transfer functions are of interest:

$$G(s) = \frac{K}{(\tau_e s + 1)(\tau_m s + 1)}$$

and

$$D(s) = \frac{1}{\tau_t s + 1}.$$

The output y is the speed of the motor and the goal is to track the reference signal r. The loop transfer function is $L(s) = D(s)G(s)$.



Figuur 1: Blokscheme of the closed loop system

step

a). Take $\tau_t = 1$, $\tau_e = 3$ and $\tau_m = 0.4$. Compute the possible values for the gain K such that the steady state error for the velocity is less than 10 percent. Motivate your answer!

$$E(s) = \frac{1}{1 + D(s)G(s)} R(s) = \frac{1}{1 + \frac{K}{(s+1)(3s+1)(0.4s+1)}} R(s)$$

5

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{1}{1 + \frac{K}{(s+1)(3s+1)(0.4s+1)}} = \frac{1}{1+K}$$

$$\frac{1}{1+K} < 0.1 \Rightarrow K > 9$$

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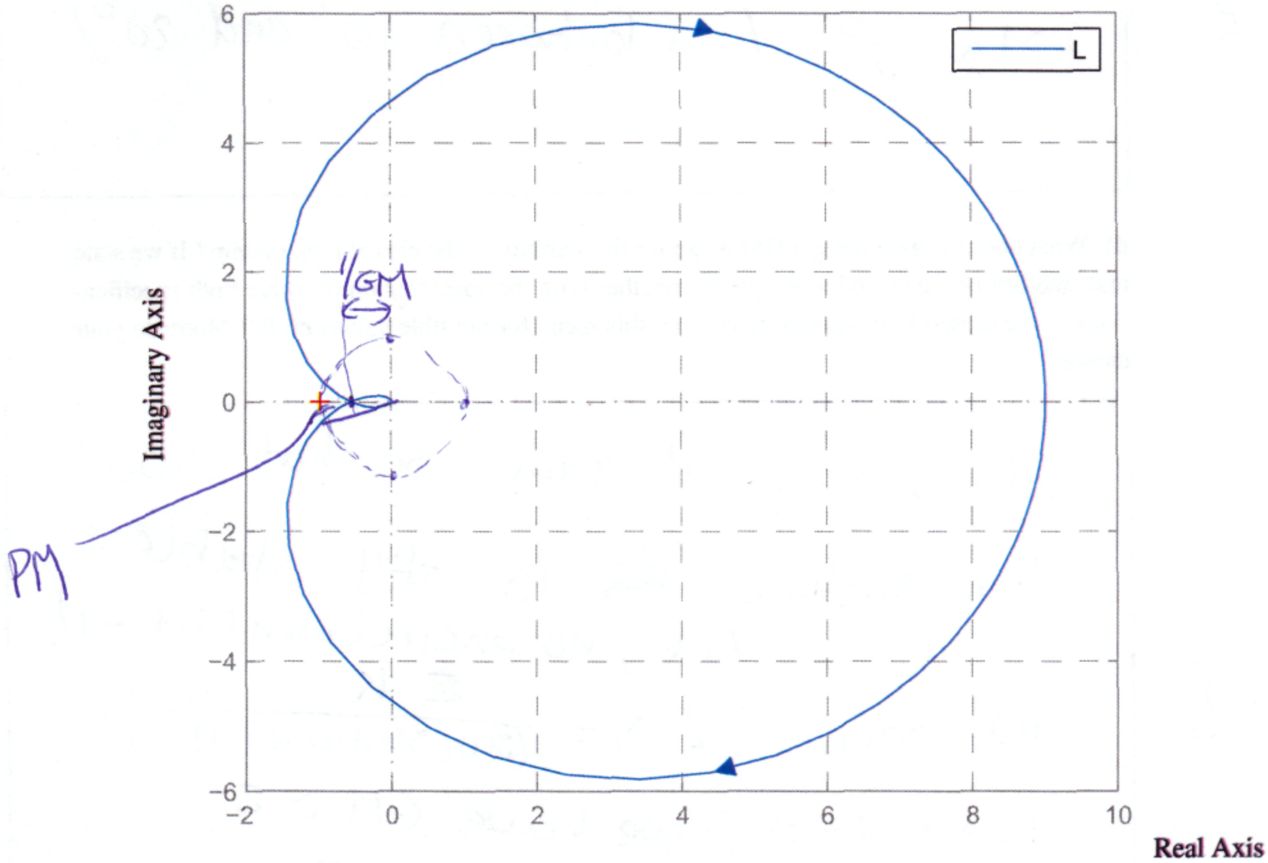
b). The Nyquist plot of L with the minimal K value of question a) is given in the figure below. Determine with help of the Nyquist criterium if the closed loop system is stable. Give Z, N and P.



Figuur 2: Nyquist plot $L(s)$

$L(s) = G(s)D(s) = \dfrac{K}{(s+1)(3s+1)(0.4s+1)}$ poles in LHP hence $L(s)$ stable

5 Hence $P=0$. No encirclements of -1, thus $N=0$. Conclusion: $Z=0$, i.e., closed loop poles are in LHP, hence closed loop stable.

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c). Give an estimate of the gain margin (GM) and phase margin (PM). Point out in the plot how you obtain them.

5

$1/GM \approx \frac{1}{2} \Rightarrow GM \approx 2$

$PM \approx 15^\circ$ (in between 10° and 20°)

d). What does the gain margin GM mean for the stability of the closed loop system? If we state that stability of the closed loop system together with the specification of a) are both specifications of the closed loop system, what does this mean for possible values of K? Motivate your answer!

6

GM is maximal gain so that closed loop system is still stable (i.e., no encirclement of -1).

We have $L(s) = \frac{K}{(s+1)(3s+1)(0.4s+1)}$

With $K=9$, we have $GM \approx 2$

Hence $9 < K < 18$

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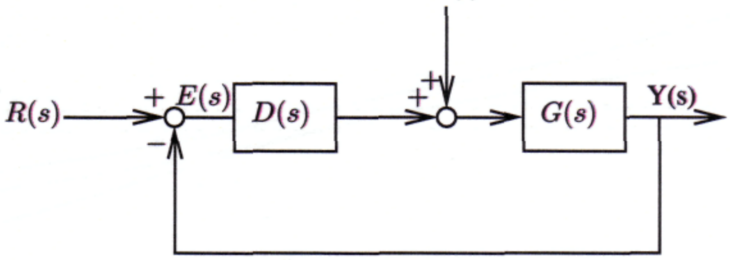
Opgave 5 (20 punten)

Given the open loop system with transfer function $G(s) = \dfrac{2}{s(s+1)}$.

We design a PID controller $D(s) = \frac{K_p}{s}(1 + T_d s)(s + \frac{1}{T_i})$ in the configuration as given in the figure below.



Figuur 3: Blokscheme of the closed loop system

a). Give the transfer function of the disturbance $W(s)$ to the output $Y(s)$. Does the chosen $D(s)$ yield a steady state error as a consequency of a constant disturbance w? Motivate your answer!

$$Y(s) = \frac{2}{s^2+s}\left(W(s) - D(s)Y(s)\right)$$

$$\Rightarrow \quad Y(s) = \frac{2}{s^2+s+2D(s)} W(s)$$

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$$y_{ss} = \lim_{s \to 0} \frac{2}{s^2+s+2D(s)} = \frac{1}{D(0)} \Rightarrow 0$$

due to integrator.

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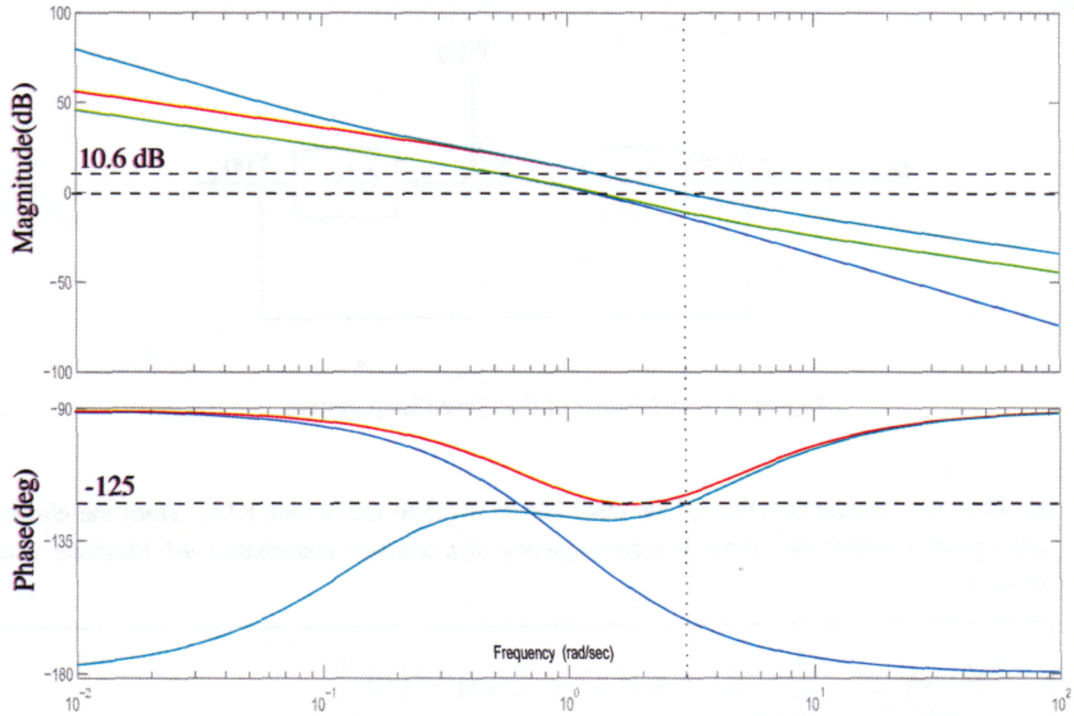
The Bode plot of the different systems is given below. The dark blue plot is the Bode plot of $G(s)$ (for low frequencies it equals the green one). The green plot, which has the same phase plot as the red one, is the Bode plot of $G(s)D(s)$ with $D(s)$ only a PD control and $K = 1$. The light blue plot, which has the same aplitude as the red one for high frequencies, is the Bode plot of the final design $G(s)D(s)$ with $D(s)$ the PID control.



b). Explain the design of $D(s)$ with help of the Bode plots. Explain the red plot. Can you determine the different specifications (like phase margin, cross over frequency,....) for the design from the plot? Give an estimate of T_d, T_i and K. Motivate your answer!

D_{red} is Bode plot $G(s)D_{red}(s)$ with a PD control,

$D_{red}(s) = K_p(1 + T_d s)$.

D_{red} shifts cross over frequency by K_p to $\omega_c \approx 3$

$D_{green}(s) = 1 + T_d s$

Since $20 \log K_p = 10.6$, we obtain

$K_p = 3.39$

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From the red and green plots we find the break frequencies approximately at $\omega \approx 3$, hence $T_d \approx 0.3$

Finally $\frac{1}{T_i} \approx 0.15 \Rightarrow T_i \approx 6.7$
(can also be ~~seen~~ from figure).

10 Final design: $D_{light blue}(s) = \frac{3.39}{s}(1+0.3s)(s+0.15)$

3 Specifications closed loop:

$\omega_c = 3$, $PM \approx 55^\circ$

END EXAM